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The solution of two wave-diffraction problems

D. PORTER and K.-W.E. CHU

Department of Mathematics, University of Reading, P.O. Box 220, Whiteknights, Reading RG6 2AX, Berkshire, England

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Summary

A method for obtaining the numerical solution of first-kind integral equations with the Hankel-function kernel $H_0^{(1)}(k|x-t|)$ is described in relation to two water-wave diffraction problems. The principal feature is the implementation of a new technique for transforming the given equations into second-kind integral equations, which have continuous kernels and from which numerical approximations can readily be determined.

1. Introduction

We describe here a new method of solving two related wave-diffraction problems which was developed to provide an efficient numerical solution, particularly in the context of coastal engineering. The technique is an improvement (in terms of computational effort) on those currently employed in this area (see, for example, Gilbert and Brampton [3]) for the two prototype problems considered, and it is capable of extension to other problems.

The standard integral equations for the two problems, which are given in the following section, are reduced to forms more suited to rapid computational techniques by an analytic procedure which falls into two essentially separate parts. In the first of these it is shown that the two integral equations, whose solutions each depend on two physical parameters, are solved via a single first-kind integral equation involving only one parameter. This simplification was first noticed by Williams [9] and in Section 2 we derive those particular relationships which form an essential part of overall procedure, by a method different from that used by Williams.

The second element of the reduction, which is the main feature of the method, involves the conversion of the simplified first-kind equation, which has a logarithmically singular kernel, into a pair of second-kind integral equations with continuous kernels. This is achieved by means of an operator, recently devised by Porter [6], which transforms the Hankel-function kernel into a Cauchy singular kernel whose principal part is easily isolated and inverted. The mechanism used for changing the kind of the integral equation can be implemented without the preliminary removal of a parameter, at the expense of a reduction in the overall efficiency, and can be applied in more general circumstances than those encountered here.

The advantages of a second-kind integral equation over one of the first kind, from the computational point of view, are well-known and various conversion methods have been developed, particularly with regard to the Hankel-function kernel (see, for example, Wickham [8]). The procedure described here appears to be new, however, and is distinguished by the fact that the kernels of the derived second-kind equations can be evaluated explicitly. Although the form of these kernels is rather complicated, the implementation of a standard numerical routine proves to be straightforward and provides accurate solutions very rapidly.

It should be remarked that the second-kind equations which are derived provide a convenient means of obtaining approximate analytic solutions (for long or short waves) but this aspect is not pursued here.

2. The reduction of the integral equations

The diffraction of a plane wave train through a gap in an infinite straight breakwater requires the solution of the integral equation

$$(Kv_\alpha)(x) = f_\alpha(x), \quad |x| < 1, \tag{2.1}$$

where

$$(Kv_\alpha)(x) \equiv \tfrac{1}{2}i\int_{-1}^{1} H_0^{(1)}(k|x-t|)v_\alpha(t)\,\mathrm{d}t,$$

$H_0^{(1)}(x)$ denoting the Hankel function of order zero, and

$$f_\alpha(x) = \exp(-ikx\cos\alpha).$$

Here k represents the (non-dimensionalised) assigned wave number and $\alpha \in [0, \pi]$ is the given incident wave angle. The unique solution of (2.1) is such that

$$v_\alpha(x) \sim (1-x^2)^{-1/2}, \quad |x| \to 1, \tag{2.2}$$

this behaviour being induced by the logarithmic singularity in the kernel of K.

The complementary problem of wave diffraction by a finite strip leads to the integro-differential equation

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + k^2\right)(K\phi_\alpha)(x) = ik\sin\alpha f_\alpha(x), \quad |x| < 1, \tag{2.3}$$

whose solution must satisfy

$$\phi_\alpha(\pm 1) = 0, \tag{2.4}$$

and it is not difficult to deduce from (2.1) and (2.2) that

$$\phi_\alpha(x) \sim (1-x^2)^{1/2}, \quad |x| \to 1. \tag{2.5}$$

In terms of the inner product

$$(f, g) = \int_{-1}^{1} f(x)\overline{g(x)}\,\mathrm{d}x,$$

quantities of prime physical interest associated with (2.1) and (2.3), the far-field diffraction coefficients, are given respectively by

$$G(\alpha, \theta) = (v_\alpha, f_{\pi-\theta}), \quad F(\alpha, \theta) = ik \sin\theta(\phi_\alpha, f_{\pi-\theta}), \tag{2.6}$$

in which $\theta \in [0, \pi]$ is the observation angle. The reciprocal relation

$$G(\alpha, \theta) = G(\theta, \alpha) \tag{2.7}$$

follows directly from (2.1) and (2.6). Further, it transpires that we ultimately need only to calculate $G(\alpha, 0)$, for which we use the shortened notation

$$G_\alpha = G(\alpha, 0) = G(0, \alpha). \tag{2.8}$$

Other symmetry properties, which can be deduced immediately from (2.1) and (2.3), are

$$v_\alpha(x) = v_{\pi-\alpha}(-x), \quad \phi_\alpha(x) = \phi_{\pi-\alpha}(-x), \quad |x| < 1, \tag{2.9}$$

and these lead to

$$G(\alpha, \theta) = G(\pi-\alpha, \pi-\theta), \quad F(\alpha, \theta) = F(\pi-\alpha, \pi-\theta). \tag{2.10}$$

Interchange of the differential and integral operators, followed by an integration by parts and use of (2.4), establishes that

$$\left(\frac{\mathrm{d}}{\mathrm{d}x} - ik\right)(K\phi_\alpha)(x) = \{K(\phi'_\alpha - ik\phi_\alpha)\}(x).$$

When this relationship is employed in (2.3) and the remaining factor in the differential operator there is removed by integration, we find that

$$\{K(\phi'_\alpha - ik\phi_\alpha)\}(x) = \cot(\tfrac{1}{2}\alpha)(f_\alpha(x) - c_\alpha f_0(x)), \quad |x| < 1,$$

for $\alpha \in (0, \pi)$. On referring to (2.1), (2.2) and (2.5) we deduce that

$$\phi'_\alpha(x) - ik\phi_\alpha(x) = \cot(\tfrac{1}{2}\alpha)(v_\alpha(x) - c_\alpha v_0(x)), \quad |x| < 1.$$

The constant c_α is determined by noting that, according to (2.4),

$$(\phi'_\alpha - ik\phi_\alpha, f_\pi) = 0,$$

which implies that

$$(v_\alpha - c_\alpha v_0, f_\pi) = 0,$$

giving

$$c_\alpha = G_0^{-1} G_\alpha$$

in the notation (2.8).

We now have an explicit connection between $\phi_\alpha(x)$ and $v_\alpha(x)$, namely

$$G_0(\phi'_\alpha(x) - ik\phi_\alpha(x)) = \cot(\tfrac{1}{2}\alpha)(G_0 v_\alpha(x) - G_\alpha v_0(x)),\ |x| < 1, \tag{2.11}$$

and the remaining analysis follows from this and the symmetries inherent in the governing integral equations which we have already noted. Thus, using (2.9) in (2.11) we have

$$G_0(\phi'_\alpha(x) + ik\phi_\alpha(x)) = -\tan(\tfrac{1}{2}\alpha)(G_0 v_\alpha(x) - G_{\pi-\alpha} v_\pi(x)),\ |x| < 1. \tag{2.12}$$

Eliminating $v_\alpha(x)$ between (2.11) and (2.12) and solving the resulting differential equation for $\phi_\alpha(x)$ in accordance with (2.4) yields

$$2G_0\phi_\alpha(x) = \sin\alpha f_\alpha(x)\int_x^1 \{G_\alpha v_0(t) - G_{\pi-\alpha} v_\pi(t)\} f_{\pi-\alpha}(t)\,\mathrm{d}t,\ |x| < 1, \tag{2.13}$$

which satisfies the condition $\phi_\alpha(-1) = 0$ identically. Either (2.11) or (2.12) may now be used to provide $v_\alpha(x)$ in the form

$$G_0 v_\alpha(x) = \cos^2(\tfrac{1}{2}\alpha) G_\alpha v_0(x) + \sin^2(\tfrac{1}{2}\alpha) G_{\pi-\alpha} v_\pi(x) - ikG_0 \sin\alpha\phi_\alpha(x),\ |x| < 1, \tag{2.14}$$

in which $\phi_\alpha(x)$ represents the function defined by (2.13).

On the basis of these expressions it is a straightforward matter to evaluate the diffraction coefficients defined by (2.6) and find that

$$2G_0(\cos\alpha + \cos\theta)F(\alpha, \theta) = \sin\alpha\sin\theta(G_{\pi-\alpha}G_{\pi-\theta} - G_\alpha G_\theta), \tag{2.15}$$

and

$$G_0 G(\alpha, \theta) = \cos^2(\tfrac{1}{2}\alpha) G_\alpha G_\theta + \sin^2(\tfrac{1}{2}\alpha) G_{\pi-\alpha}G_{\pi-\theta} - \operatorname{cosec}\theta\sin\alpha G_0 F(\alpha, \theta). \tag{2.16}$$

Since

$$v_\pi(x) = v_0(-x) \tag{2.17}$$

by (2.9), we conclude from the foregoing relations that $\phi_\alpha(x)$, $v_\alpha(x)$, $F(\alpha, \theta)$ and $G(\alpha, \theta)$ are completely determined by merely solving

$$(Kv_0)(x) = f_0(x),\ |x| < 1, \tag{2.18}$$

for $v_0(x)$ and evaluating

$$G_\alpha = (v_0, f_{\pi-\alpha}) \tag{2.19}$$

for a selection of values of α, where $0 \leqslant \alpha \leqslant \pi$.

3. The integral equations for $v_0(x)$

It is convenient at this point to decompose $v_0(x)$ into its even and odd parts and, using (2.17), to set

$$v_0(x)=v_s(x)+v_a(x),\ v_\pi(x)=v_s(x)-v_a(x), \tag{3.1}$$

where

$$2v_s(x)=v_0(x)+v_0(-x),\ 2v_a(x)=v_0(x)-v_0(-x).$$

In consequence, (2.18) is replaced by the two integral equations

$$\left.\begin{aligned}&\int_0^1 v_s(t)\{H_0^{(1)}(k|x-t|)+H_0^{(1)}(k|x+t|)\}\,\mathrm{d}t=-2i\cos(kx),\\&\int_0^1 v_a(t)\{H_0^{(1)}(k|x-t|)-H_0^{(1)}(k|x+t|)\}\,\mathrm{d}t=2\sin(kx),\end{aligned}\right\}\quad 0\leqslant x<1. \tag{3.2}$$

Our approach is to first transform each of these equations into a Cauchy singular equation of the first kind and then, by inverting the principal part, to deduce a pair of second-kind equations with regular kernels from which $v_s(x)$ and $v_a(x)$ may be determined numerically. The former objective is achieved by making use of the operator M introduced by Porter [6] and defined by

$$(M\phi)(x)\equiv\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2}+k^2\right)\int_0^x J_0(k(x-t))\phi(t)\,\mathrm{d}t,\ x>0,$$

for a function $\phi(x)$ integrable on a given interval. The particular property of M which is significant here was derived in the earlier work and is as follows. If we denote

$$h_\pm(x,t)=H_0^{(1)}(k|x\pm t|),$$

where t is regarded as fixed, then

$$(Mh_\pm)(x)=\mp kt\{J_0(kx)H_1^{(1)}(kt)\pm J_1(kx)H_0^{(1)}(kt)\}/(t\pm x),$$

for $x\neq\mp t$. Here we have used the standard notation for Bessel functions.

Recalling that $f_0(x)=\exp(-ikx)$, it is easy to establish that

$$(Mf_0)(x)=-kJ_1(kx)+ikJ_0(kx),$$

and we may therefore explicitly express the effect of applying M to the equations (3.2).

After some rearrangement and use of the Wronskian relation for Bessel functions, we arrive at

$$\left.\begin{aligned}\frac{1}{\pi i}\int_0^1 \frac{2xv_s(t)}{t^2-x^2}\,\mathrm{d}t &= ikJ_1(kx)-\int_0^1 N_s(x,\,t)v_s(t)\,\mathrm{d}t,\\ \frac{1}{\pi i}\int_0^1 \frac{2tv_a(t)}{t^2-x^2}\,\mathrm{d}t &= -kJ_0(kx)-\int_0^1 N_a(x,\,t)v_a(t)\,\mathrm{d}t,\end{aligned}\right\}\ 0\leqslant x<1. \tag{3.3}$$

in which the kernels

$$\begin{aligned}N_s(x,\,t) &= ktH_0^{(1)}(kt)\frac{[tJ_1(kx)-xJ_1(kt)]}{t^2-x^2}-ktxH_1^{(1)}(kt)\frac{[J_0(kt)-J_0(kx)]}{t^2-x^2},\\ N_a(x,\,t) &= ktH_0^{(1)}(kt)\frac{[tJ_1(kt)-xJ_1(kx)]}{t^2-x^2}-kt^2H_1^{(1)}(kt)\frac{[J_0(kt)-J_0(kx)]}{t^2-x^2},\end{aligned} \tag{3.4}$$

are regular.

The integrals on the left of (3.3) are to be interpreted in the sense of Cauchy principal values, and the corresponding operators can be inverted by means of the method developed by Muskhelishvili [5]. We require solutions for $v_{s,a}(x)$ which are bounded at $x=0$ (with $v_a(0)=0$) and unbounded at $x=1$, in accordance with (2.2). In fact we may at this stage remove the end-point singularities by introducing new functions $\psi_{s,a}(x)$, continuous in [0, 1], such that

$$\psi_{s,a}(x)=(1-x^2)^{1/2}v_{s,a}(x),\ 0\leqslant x\leqslant 1. \tag{3.5}$$

Having solved (3.3), regarding the right-hand sides as known, and implemented (3.5), the resulting integral equations may be written jointly in the form

$$\psi_{s,a}(x)=F_{s,a}(x)+\int_0^1 l_{s,a}(x,\,t)\psi_{s,a}(t)\,\mathrm{d}t,\ 0\leqslant x\leqslant 1. \tag{3.6}$$

The kernels occurring here are

$$\begin{aligned}l_s(x,\,t) &= \frac{2i}{\pi}\int_0^1 \frac{u(1-u^2)^{1/2}}{(1-t^2)^{1/2}}\,\frac{N_s(u,\,t)}{u^2-x^2}\,\mathrm{d}u,\\ l_a(x,\,t) &= \frac{2i}{\pi}\int_0^1 \frac{x(1-u^2)^{1/2}}{(1-t^2)^{1/2}}\,\frac{N_a(u,\,t)}{u^2-x^2}\,\mathrm{d}u,\end{aligned} \tag{3.7}$$

and the free terms are given by

$$\begin{aligned}F_s(x) &= \frac{2k}{\pi}\int_0^1 \frac{t(1-t^2)^{1/2}J_1(kt)}{t^2-x^2}\,\mathrm{d}t+a_s,\\ F_a(x) &= \frac{2ki}{\pi}\int_0^1 \frac{x(1-t^2)^{1/2}J_0(kt)}{t^2-x^2}\,\mathrm{d}t.\end{aligned} \tag{3.8}$$

The appearance of the constant a_s, indeterminate at this stage, indicates that the equation for $\psi_s(x)$ is not equivalent to the original first-kind equation for $v_s(x)$. The equations which determine $v_a(x)$ and $\psi_a(x)$ are equivalent however.

This technique for regularising a singular integral equation is standard, but its usefulness in practical terms depends on convenient forms being available for the elements of the new equations. In the present case, the integrals (3.7) and (3.8) can be evaluated by using the Fourier expansions

$$J_0(kx) = J_0^2(\tfrac{1}{2}k) + 2\sum_{n=1}^{\infty}(-1)^n J_n^2(\tfrac{1}{2}k) T_{2n}(x),$$

$$J_1(kx) = 2\sum_{n=0}^{\infty}(-1)^n J_n(\tfrac{1}{2}k) J_{n+1}(\tfrac{1}{2}k) T_{2n+1}(x),$$

given by Luke [4], in conjunction with the standard integral

$$\frac{1}{\pi}\int_{-1}^{1}\frac{T_n(t)\,\mathrm{d}t}{(1-t^2)^{1/2}(t-x)} = U_{n-1}(x),\ |x|<1,$$

where $T_n(x)$ and $U_n(x)$ are the first- and second-kind Chebyshev polynomials, respectively.

Before setting down the expressions which result, it is convenient to make the variable change

$$x = \cos\theta,\ \psi_{s,a}(\cos\theta) = \Psi_{s,a}(\theta),\ 0 \leqslant \theta \leqslant \tfrac{1}{2}\pi, \tag{3.9}$$

following which the final form of the second-kind equations is

$$\Psi_{s,a}(\sigma) = h_{s,a}(\sigma) + (K_{s,a}\Psi_{s,a})(\sigma),\ 0 \leqslant \sigma \leqslant \tfrac{1}{2}\pi. \tag{3.10}$$

The kernels of the operators

$$(K_{s,a}\Psi_{s,a})(\sigma) \equiv \int_0^{\pi/2} k_{s,a}(\sigma, \theta)\Psi_{s,a}(\theta)\,\mathrm{d}\theta$$

can be arranged in the following form which proves convenient for the subsequent computations:

$$\begin{aligned} k_s(\sigma, \theta) &= ik\cos\theta\{H_1^{(1)}(k\cos\theta)S_1(\sigma, \theta) - \cos\theta H_0^{(1)}(k\cos\theta)S_3(\sigma, \theta)\},\\ k_a(\sigma, \theta) &= ik\cos\theta\{H_1^{(1)}(k\cos\theta)S_2(\sigma, \theta) - \cos\sigma H_0^{(1)}(k\cos\theta)S_3(\sigma, \theta)\}, \end{aligned} \tag{3.11}$$

where

$$S_1(\sigma, \theta) = -\sum_{n=2}^{\infty}(-1)^n\left[J_{n+1}^2(\tfrac{1}{2}k) - J_{n-1}^2(\tfrac{1}{2}k)\right]S_n(\theta-\sigma)S_n(\theta+\sigma),$$

$$S_2(\sigma, \theta) = -\sum_{n=1}^{\infty} (-1)^n \left[J_{n+1}^2(\tfrac{1}{2}k) + J_n^2(\tfrac{1}{2}k)\right]\left[S_n(\theta-\sigma)S_{n+1}(\theta+\sigma)\right.$$

$$\left. + S_{n+1}(\theta-\sigma)S_n(\theta+\sigma)\right], \quad (3.12)$$

$$S_3(\sigma, \theta) = \frac{8}{k}\sum_{n=1}^{\infty} (-1)^n n J_n^2(\tfrac{1}{2}k) S_n(\theta-\sigma)S_n(\theta+\sigma),$$

and $S_n(\theta) = \sin(n\theta)/\sin\theta$. The free terms in (3.10) are given by

$$\begin{aligned} h_s(\sigma) &= C_s + 4\sum_{n=0}^{\infty} (-1)^n n J_n^2(\tfrac{1}{2}k)\cos(2n\sigma), \\ h_a(\sigma) &= -ik\sum_{n=1}^{\infty} (-1)^n \left[J_{n+1}^2(\tfrac{1}{2}k) + J_n^2(\tfrac{1}{2}k)\right]\cos(2n+1)\sigma, \end{aligned} \quad (3.13)$$

and we have accumulated the constants arising in the equation for $\Psi_s(\sigma)$ (including a_s) in C_s. This constant is determined by referring to the original equation for $v_s(x)$, evaluated at $x = 0$, which in terms of the current variables requires that

$$\int_0^{\pi/2} \Psi_s(\theta) H_0^{(1)}(k\cos\theta)\, d\theta = -i. \quad (3.14)$$

If we decompose $\Psi_s(\sigma)$ in the form

$$\Psi_s(\sigma) = \Psi_s^{(1)}(\sigma) + C_s\Psi_s^{(2)}(\sigma), \quad (3.15)$$

where

$$\left.\begin{aligned} \Psi_s^{(1)}(\sigma) &= \left(h_s(\sigma) - C_s\right) + \left(K_s\Psi_s^{(1)}\right)(\sigma), \\ \Psi_s^{(2)}(\sigma) &= 1 + \left(K_s\Psi_s^{(2)}\right)(\sigma), \end{aligned}\right\} 0 \leqslant \sigma \leqslant \pi/2, \quad (3.16)$$

then, following the solution of these last two equations, C_s is given explicitly by using (3.15) in (3.14).

4. The numerical procedure and results

Despite the complicated forms of the two integral equations (3.10) to which the original problems have been reduced, their numerical solution is straightforward. It should be noted that the kernels (3.11) are continuous for $0 \leqslant \sigma, \theta \leqslant \frac{1}{2}\pi$. Their first derivatives are also continuous in the same region except for a logarithmic singularity in $\partial k_a/\partial\theta$ at $\theta = \frac{1}{2}\pi$, the effect of which is minimised by the fact that $\Psi_a(\theta)$ vanishes at this point. On the basis of this smoothness it is anticipated that fairly accurate solutions of (3.10) can be obtained by using a relatively crude quadrature scheme to approximate the integrals, and this proves to be the case. As a result the numerical evaluation of the kernels is necessary at comparatively few points.

The forms of the kernels given in (3.11) are not the most concise available, but they are the most convenient on which to base computations. The function $S_n(\theta)$ is calculated by means of the recursion formula

$$S_{n+1}(\theta) = S_n(\theta)\cos\theta + \cos(n\theta),\quad n = 1, 2, \ldots,$$

with $S_1(\theta) = 1$, and the Bessel functions can be computed to any required accuracy using the stable recurrence routines given in Abramowitz and Stegun [1]. Bearing in mind that we are only concerned with values of k which are $O(1)$ (for which neither the short- or long-wavelength asymptotic solutions of the diffraction problems are appropriate and the numerical solution is essential), the summations in (3.12) and (3.13) converge rapidly and their values can accurately be found by truncation. Terms of the form $\cos\theta H_1^{(1)}(k\cos\theta)$ are evaluated at $\theta = \frac{1}{2}\pi$ by using the appropriate asymptotic expansion of the Hankel function.

The numerical routine selected to solve the integral equations is the Nyström method used in conjunction with the composite Gauss-Legendre rule ${}_mG_n$ (that is, the n-th order Gauss-Legendre rule applied on m equal subintervals). This procedure compares favourable with competing methods (see Baker [2] and Riddell and Delves [7]).

The quantities of principal interest can be calculated from the approximate solutions of (3.10) by first using (3.5) and (3.1) to produce $v_0(x)$ and $v_\pi(x)$. The numerical integration of G_α as given by (2.19) and subsequently of the function $\phi_\alpha(x)$ defined by (2.13) is performed using interpolation where necessary. We can finally construct $v_\alpha(x)$ according to (2.14) and the diffraction coefficients given by (2.15) and (2.16). We note here that

Table 1. The far-field coefficients $|G(\alpha, \theta)|$ (upper values) and $|F(\alpha, \theta)|$ (lower values) for $k = \frac{1}{2}\pi$

θ	α				
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
0	0.99 –	0.98 –	0.99 –	1.08 –	1.67 –
$\pi/6$	0.98 –	1.00 0.60	1.07 0.92	1.26 1.25	2.01 1.67
$\pi/4$	0.99 –	1.07 0.92	1.23 1.43	1.54 1.93	2.39 2.55
$\pi/3$	1.08 –	1.26 1.25	1.54 1.93	1.95 2.59	2.80 3.36
$\pi/2$	1.67 –	2.01 1.67	2.39 2.55	2.80 3.36	3.25 4.18
$2\pi/3$	2.43 –	2.66 1.47	2.87 2.21	3.02 2.84	2.80 3.36
$3\pi/4$	2.67 –	2.79 1.17	2.88 1.73	2.87 2.21	2.39 2.55
$5\pi/6$	2.78 –	2.82 0.80	2.79 1.17	2.66 1.47	2.01 1.67
π	2.84 –	2.78 –	2.67 –	2.43 –	1.67 –

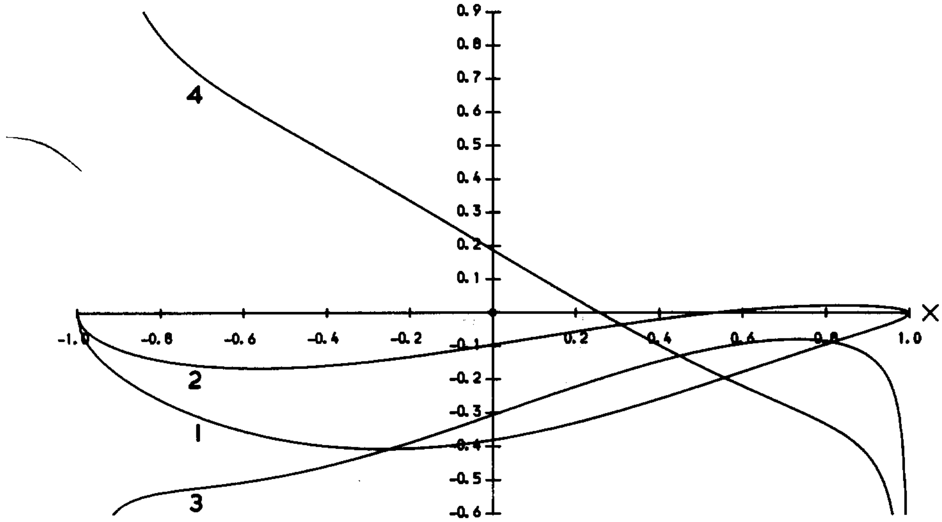

Figure 1. Graphs of 1: Re($\phi_\alpha(x)$), 2: Im($\phi_\alpha(x)$), 3: Re($v_\alpha(x)$), 4: Im($v_\alpha(x)$) for $\alpha = \frac{1}{4}\pi$ and $k = \frac{1}{2}\pi$.

$F(\alpha, \pi - \alpha)$ is obtained via an expression which is derived from (2.15) by taking the appropriate limit, and which involves $\mathrm{d}G_\alpha/\mathrm{d}\alpha$, a quantity readily computed using (2.19).

A more efficient routine arises if (3.1) and (3.5) are explicitly used in (2.19), (2.13) and (2.14), for then the fact that $\psi_{s,a}(x)$ are even, odd functions of x implies that duplication in the computations is avoided.

Either way the specimen values of the (real) diffraction coefficients $|G(\alpha, \theta)|$ and $|F(\alpha, \theta)|$ shown in Table 1 are obtained. Where comparison is possible, these are found to be in good agreement with the results of Gilbert and Brampton [3]. We also include, in Figure 1, graphs of $\phi_\alpha(x)$ and $v_\alpha(x)$ for the particular value $\alpha = \pi/4$ of the incident wave angle.

Numerical experiments have shown that two decimal places of accuracy is achieved with $m = n = 4$ in the quadrature rule. With these values, the calculation of the quantities in Table 1, and of $\phi_\alpha(x)$ and $v_\alpha(x)$ for eight values of α, is completed in 2 seconds of computer time on a NORD-500 system. This suggests that extensions of the method we have used, particularly of the first- to second-kind integral-equation transformation, could provide extremely effective routines for solving more complicated diffraction problems.

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